Determinants of Modal Choice Behavior Using Nested Logit Model: A Journey to Work Trip in Dhaka City

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Introduction

In recent years, urban policymakers faced with the growing and complex problems of air pollution and congestion have begun to ask for more sophisticated decision-making tools, including models to forecast travel demand and its effect under various circumstances. Discrete choice models have played an important role in transportation modeling for the last 30 years. They are namely used to provide a detailed representation of the complex aspects of transportation demand, based on strong theoretical justifications. The art of finding the appropriate model for a particular application requires from the analyst both a close familiarity with the reality under interest and a strong understanding of the methodological and theoretical background of the model.

This paper describes the development of mode choice model for Dhaka city considering journey to work trip. This mode choice model uses the following attributes: travel time, travel cost, travel time variability as level of service and socio-economic characteristics such as age, gender, income and occupation to estimate the proportions of trips which is composed of motorized vehicles (Auto(car/taxi), Transit(bus) and CNG) or non-motorized vehicles (rickshaw and walk). The mode choice model was calibrated using the nested logit model formulation.

This calibration used trip records from 100 samples of Dhaka city in August – September, 2009. The calibration of the modal choice model was performed using the computer software LIMDEP (LIMited DEPendent variable modeling) which is now widely used by the researchers in universities, government sectors and industry for data analysis with modeling concept. This program allows the user to calibrate various type of discrete choice model such as multinomial logit mode, nested logit model, probit model etc.

The adopted structure of this nested logit model consists of a two-level nesting structure as illustrated in Figure 1. In the primary nest, total person trips were divided into motorized vehicle (MV) trips, and non-motorized vehicle (NMV) trips. In the second nest, the MV trips were split into auto (car/taxi), transit (bus) and CNG and the NMV trips were split into rickshaw and walk. However, the model was validated to ensure that the model replicated observed shares.

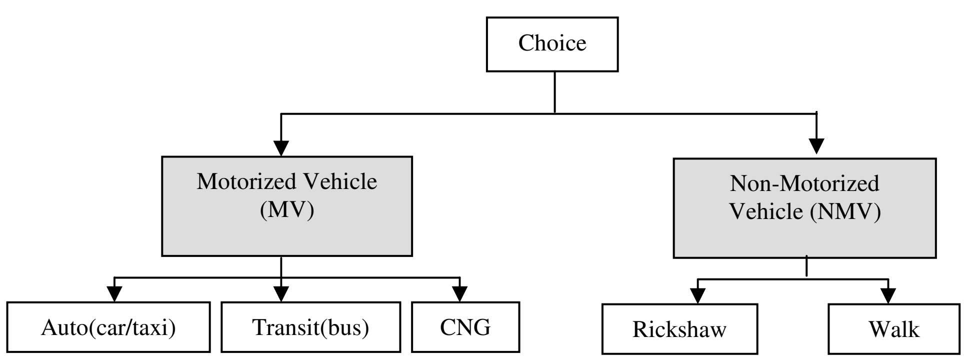

Fig. 1: Two-level nested structure for five alternatives.

Literature Review

For modeling discrete choice decisions, e.g. mode choice, in the context of random utility theory usually the multinomial logit model (MNL) (Guadagni and Little, 1998) is used though it has some well known limitations (McFadden, 1974). The MNL assumes proportional substitution patterns (Independence of Irrelevant Alternatives, IIA). To overcome this restrictive assumption, one possible alternative is to use the nested logit model for estimation in practical applications (Guadagni and Little, 1983; de Dios Ortúzar, 2001). The nested logit model admits more general substitution patterns and nevertheless remains, in contrast to the probit model as another alternative to overcome the aforementioned restrictive assumptions, analytically tractable.

Utility-based choice or choice based on the relative attractiveness of competing alternatives from a set of mutually exclusive alternatives is called a discrete choice situation. Discrete choice models are interpreted in terms of an underlying behavioral model, the so called random utility maximization (RUM) model. The decision-maker chooses the alternative with the highest utility. Characteristics of the choice alternatives and of the decision-maker determine the alternatives' utilities. The latter do not have a direct utility contribution per se, but serve as proxies for consumer heterogeneity.

Modeling discrete consumer decisions is characterized by a trade-off between flexibility and ease of the estimation (Munizaga and Alvarez-Daziano, 2001). On the one hand, probit models assume a more realistic situation by allowing a correlation structure of the error terms. However, the estimation of these models can become very complex because of the underlying multidimensional integrals. On the other hand, there are logit models which are distinguished by closed choice probabilities but, due to restrictive substitution patterns i.e. the above mentioned IIA assumption, are often not very realistic. Nevertheless, because of its ease in estimation logit models are favored. Their estimation is usually based on the multinomial logit (MNL) model. To overcome the restrictive substitution assumptions between alternatives, various extensions of the MNL exist, all with the general solution of allowing correlations between the alternatives' error terms.

The idea of the nested logit model lies in the grouping of similar alternatives into nests, creating a hierarchical structure of the alternatives (Ben-Akiva and Lerman, 1985; Train, 2003). The error terms of alternatives within a nest are correlated with each other, and the error terms of alternatives in different nests are uncorrelated. The nested logit approach is predominantly used in the field of transportation research and logistics (Train, 1980; Bhat, 1997; Knapp *et al*., 2001), but can also be appropriate for marketing issues (Kannan and Wright, 1991; Chintagunta, 1993; Chintagunta and Vilcassim, 1998; Guadagni and Little, 1998; Chib *et al*., 2004). The nested logit model can be used for modeling in any situation where subsets of alternatives share unobservable utility components (Ben-Akiva and Lerman, 1985). This is usually applied in case of choice modeling (Kamakura *et al*., 1996; Ailawadi and Neslin, 1998; Guadagni and Little, 1998; Sun *et al*., 2003; Chib *et al*., 2004). For example, in the case of urban mode choice among drive alone, shared ride, bus and light rail; the bus and light rail alternatives are likely to be more similar to each other than they are to either of the other alternatives due to shared attributes which are not included in the measured portion of the utility function; again, bus and light rail may have the same fare structure and operating policies, the same lack of privacy, control of the environment, and so on. Such similarities, if not included in the measured portion of the utility function, lead to correlation between the errors associated with these alternatives, a violation of the assumptions which underlie the derivation of the MNL (Koppelman and Bhat, 2006).

Another important point to make is that the nested logit model is a combination of standard logit models. Marginal and conditional choice decisions are combined by a nesting structure (Hensher *et al*., 2005). The only goal of this process is to accommodate the violation of the IIA-assumption.

The nested logit model differs from the standard logit model in that the error components of the choice alternatives do not necessarily need to have the same distribution. Thus the nested logit model accounts for the fact that each alternative may have specific information in its unobservable utility component, which plays a role in the decision process. Subsets of alternatives may have similar information content, such that correlations between pairs of alternatives may exist (Hensher *et al*., 2005). The classification of alternatives regarding their similarities into nests and the thus resulting tree structure does not have anything in common with a stochastic valuation of alternatives within the scope of a decision tree. Nested logit models do not define the process of decision-finding, but account for differences in variances in the unobservable utility components (Hensher *et al*., 2005).

Survey and Data Preparation

The primary data source has been used for this analysis which was collected through questionnaire survey in Dhaka city of Bangladesh from August – September, 2009. This survey included a questionnaire to be filled out by the household heads who work. For the trip makers, the level of service (time, cost, travel time variability) and socio-economic characteristics (age, gender, income and occupation) were asked.

The entire dataset was reviewed and checked to make sure that it can support estimation and calibration of mode choice model. For estimating a full mode-choice model, the following explanatory variable/attributes were used:

- Travel cost (in Taka)
- Travel time (in minutes)
- Travel time variability (in minutes)
- Income (in Taka)
- Occupation (dummy variable)
- Age (in years)
- Gender (dummy variable)

Methodology

Multinomial Logit (MNL) Model

The logit model allocates person trips to alternative modes. It does so by comparing the utilities of all alternative modes. The hypothesis underlying discrete choice models is that when faced with a choice situation, an individual's preferences toward each alternative can be described by an "attractiveness" or utility measure associated with each alternative. This utility function incorporates the attributes of the alternatives as well as the decision maker characteristics. The decision-maker is assumed to choose the alternative that yields the highest utility. Utilities, however, cannot be observed or measured directly. Furthermore, many of the attributes that influence individual's utilities cannot be observed and must therefore be treated as random. Consequently, the utilities themselves in models are random, meaning that choice models can give only the probability with which alternatives are chosen, not the choice itself.

Let $U = (U_1,\ldots,U_k)$ denote the vector of utilities associated with a given set of alternative, K. This set includes k alternatives numbered $1, 2, \ldots..K$. The utility of each alternative to a specific decision maker can be expressed as a function of the observed attributes of the alternatives and the observed characteristics of this decision maker. Let a denotes the vector of variables which include these characteristics and attributes. Thus $U_i = U_i(a)$. To incorporate the effects of unobserved attributes and characteristics, the utility of each alternative is expressed as a random variable consisting of systematic (deterministic) component, $V_K(a)$ and an additive random "error term", $\zeta_i(\theta,a)$, that is,

$$Ui(\theta,a) = Vi(\theta,a) + \zeta_i(\theta,a) \qquad \forall i \in K$$

In this context, $U_K(a)$ is sometimes referred to as the "perceived utility of alternative K by the decision maker" and $V_K(a)$ as the "measured utility of alternative K by the analyst". The measured attractiveness functions $V_i(\theta,a)$ may take any finite real values and they need not be related in any way. The random disturbances $\zeta_i(\theta,a)$ can be interpreted as capturing different things, among them, errors in the measurement of the attributes in the data and the contribution of neglected attributes (attributes that can not be observed plus attributes that, although observed, are not included in $V_i(\theta,a)$ toward $U_i(\theta,a)$.

If a joint distribution of the error terms $\zeta_i(\theta,a)$ or that of $U_i(\theta,a)$ is known and attractiveness functions are specified, it is possible to obtain the choice function by calculating the probability that alternative i is the most attractive:

$$P_i(\theta,a) = Pr\,\{V_i(\theta,a) + \zeta_i(\theta,a) > V_j(\theta,a) + \zeta_j(\theta,a);\ \forall j \neq i\} \quad \forall i,j \in K$$

McFadden (1974) modeled ζ by a set of independent identically distributed Gumbel variants, with zero mean and independent of θ and a. Then, the multinomial logit model (MNL) is as follow:

$$P_n(i) = \frac{e^{\beta_i X_n}}{\sum_I e^{\beta_i X_n}} \qquad i = 1, 2, \ldots\ldots I$$

where $P_n(i)$ is the probability that person n chooses mode i, x_n is a vector of measurable characteristics of the trip maker n, and β_i is a vector of estimable coefficients by standard maximum likelihood methods.

One of the most widely discussed aspects of the multinomial logit model is the independence from irrelevant alternatives property, or IIA. The IIA property holds that for a specific driver the ratio of the choice probabilities of any two modes is entirely unaffected by any other alternatives. The IIA property is a result of the assumption that the disturbance terms are mutually independent. The IIA can be easily shown to hold in the case of MNL as follows:

$$P_n(i) / P_n(j) = \left(\frac{e^{\beta_i X_n}}{\sum_I e^{\beta_i X_n}}\right) / \left(\frac{e^{\beta_j X_n}}{\sum_I e^{\beta_i X_n}}\right) = \frac{e^{\beta_i X_n}}{e^{\beta_j X_n}} = e^{(\beta_i - \beta_j) X_n}$$

Hausman and McFadden (1984) investigated a wide range of computationally feasible tests to detect violations of the IIA assumption. This involves comparisons of logit models estimated with subsets of alternatives from the universal choice set. If the IIA assumption holds for the full choice set, then the logit model also applies to a choice from any subset of alternatives. Thus, if the logit model is correctly specified, consistent coefficient estimates of the same sub-vector of parameters from a logit model estimated with the full choice set and from a logit model estimated with a restricted choice set can be obtain.

Nested Logit (NL) Mode Choice Model

One way to relax the homoscedastiticy assumption (i.e., equal variances of distributions of errors) in the multinomial logit model that provides an intuitively appealing structure is to group the alternatives into subgroups that allow the variance to differ across the groups while maintaining the IIA assumption within the group. This specification defines a nested logit model. The nested logit model is currently the preferred extension to the simple multinomial logit discrete choice model. The appeal of the nested logit model is its ability to accommodate differential degrees of interdependence (i.e. similarity or dissimilarity) between subsets of alternatives in a choice set. In this paper, a general outline of nested logit model has been also demonstrated.

A nested logit structure allows estimation of proportions among selected sub-modes, prior to the estimation of proportions between modes. For examples, a nested logit model might estimate the proportions between motorized and non-motorized vehicle usage, prior to estimating the proportions among auto (car/taxi), transit (bus), CNG, rickshaw and walk. This ability of the nested logit model reduces some of the limitations of the multinomial logit model, specially the independence from irrelevant alternatives (IIA) limitation. It has also been found that the selection between sub-modes may be more sensitive to travel times and costs than the selection between modes.

For examples, fairly small travel time or travel cost changes can shift trips from the autos to transit much more. The nested logit structure accounts for these differences in sub-mode sensitivities to a far greater extent than a multinomial logit model. Each nest within the choice set is associated with a pseudo-utility, called composite utility, expected maximum utility, inclusive value or accessibility in the literature.

The nested logit model, first derived by Ben-Akiva (1985), is an extension of the multinomial logit model designed to capture correlation among alternatives. It is based on the partitioning of the choice set C into several nests C_K. Where, for each pair $C_K \cap C_j = 0$. The utility function of each alternative is composed of a term specific to the alternative, and a term associated with the nest. If $i \in C_K$, then

$$U_i = V_i + \varepsilon_i + V_{CK} + \varepsilon_{CK}$$

The error terms ε_i and ε_{CK} are supposed to be independent. As for the multinomial logit model, error terms (ε_i's) are supposed to be independent and identically Gumbel distributed, with scale parameter σ_k. The distribution of ε_{CK} is such that the random variable max $j \in C_K$ U_j is Gumbel distributed with scale parameter μ.

In the nested logit model the correlated alternatives are placed in a "nest", which partly removes the IIA property. There is a simple example in Figure 1 of the grouping of the alternatives. It must be noted that "motorized vehicle" is not available as an alternative because it is merely a label for a nest. It can be called "composite alternative" and the real alternatives "elemental alternatives".

To fix the idea of a nested logit model, suppose that N alternatives can be divided into M subgroups such that the choice set can be written as: $[n_1,\ldots,n_m]_m$; $m = 1,\ldots,M$ and $\sum n_m = N$. This choice-set partitioning produces a nested structure. Logically, one may think of the choice process as that of choosing among M choice sets and then making the specific choice with the chosen set. The mathematical form for a two-nested level logit model is as follows:

$$P_n = P_{n|m} P_m$$

$$P_{n|m} = \frac{exp(\beta' x_j / m)}{\sum_{n_m} exp(\beta' x_j / m)}$$

$$P_m = \frac{exp(\gamma' z_m + \theta_m I_m)}{\sum_{m} exp(\gamma' z_m + \theta_m I_m)}$$

$$I_m = ln \sum_{n_m} exp(\beta' x_j / m)$$

Where, P_n is the unconditional probability of choice n;

$P_{n|m}$ is the conditional probability of choosing alterative n given that person has selected the choice-set m;

P_m is the probability of selecting the choice-set m;

$\boldsymbol{x}_{n|m}$ are attributes of the choices;

z_m are attributes of the choice sets;

I_m is called the inclusive value (log sum) of choice-set m;

β and γ are vectors of coefficients to be estimated; and

θ_m is the coefficient of the inclusive value (Log sum) of choice-set m.

If all inclusive value parameters are restricted to be 1, then the nested logit model will be similar to multinomial logit model. The nested logit model is consistent with random utility maximization if the conditions' inclusive value parameter (θ) is bounded between zero and one.

Calibration of Mode Choice Models

The survey data was used to estimate mode choice model using a basic specification which includes travel time, travel cost, travel time variability, income, occupation, age and gender as the explanatory variables. Travel time and travel cost represent mode related attributes; all other things being equal, a faster mode of travel is more likely to be chosen than a slower mode and a less expensive mode is more likely to be chosen than a costlier mode.

The estimation results, the nested logit model reported in this study were obtained from using LIMDEP (NLOGIT 3.0) software package. The outputs from the software package typically include, at least, the following estimation results:

- Parameter names, parameter estimates, standard errors of these estimates and the corresponding t-statistics for each variable/parameter;
- Log-likelihood values at zero (equal probability model), estimated model and at convergence and
- Rho-Squared and other indicators of goodness of fit.

Goodness-of-Fit Measures

The rho-squared value (ρ^2) which can be used to describe the overall goodness of fit of the model. The rho-squared (ρ_0^2) value is based on the relationship among the log-likelihood values: *LL(0)* represents the log-likelihood with zero coefficients (which results in equal likelihood of choosing each available alternative), *LL*(C) represents the log-likelihood for the constants only model, $LL(\hat{\beta})$ represents the log-likelihood for the estimated model and *LL(*) = 0* is the log-likelihood for the perfect prediction model. It is simply the ratio of the distance between the reference model and the estimated model divided by the difference between the reference model and a perfect model. If the reference model is the equally likely model, the rho-square with respect to zero, ρ_0^2, is:

$$\rho_0{}^2 = \frac{LL(\hat{\beta}) - LL(0)}{LL(*) - LL(0)}$$

Since the log-likelihood value for the perfect model is zero, the ρ_0^2 measure reduces to:

$$\rho_0{}^2 = 1 - \frac{LL(\hat{\beta})}{LL(0)}$$

By definition, the values of both rho-squared measures lie between 0 and 1 (this is similar to the R^2 measure for linear regression models). A value of zero implies that the model is no better than the reference model, whereas a value of one implies a perfect model; that is, every choice is predicted correctly.

Statistical Tests

The t-statistic of an estimated parameter is an important statistical test. The t-statistic presented here tests the hypothesis that the true value is zero i.e. the variable has no effect on modal utilities. The rejection of this null hypothesis implies that the corresponding variable has a significant impact on the modal utilities and suggests that the variable should be retained in the model. Low absolute values of the t-statistic imply that the variable does not contribute significantly to the explanatory power of the model and can be considered for exclusion. The statistic used for testing the null hypothesis that a parameter $\hat{\beta}_k$ is equal to some hypothesized value, $\beta_k{}^*$, is the asymptotic t-statistic, which takes the following form:

$$t - statistic = \frac{\hat{\beta}_k - \beta_k{}^*}{S_k}$$

where, $\hat{\beta}_k$ is the estimate for the k^{th} parameter, $\beta_k{}^*$ is the hypothesized value for the k^{th} parameter and S_k is the standard error of the estimate.

Maximum Likelihood (FIML) Estimation

For the nested logit models, there are two ways to estimate the parameters of the nested logit model. A limited information maximum likelihood (LIML), sequential (multi-step) maximum likelihood approach can be done as follows: estimate β by treating the choice within branches as simple multinomial logit model, compute the inclusive values for all branches in the model, then estimate the parameters by treating the choice among branches as a simple multinomial logit models. Since this approach is a multi-step estimator, the estimate of the asymptotic covariance matrix of the estimates at the second step must be corrected.

The other approach of estimating a nested logit model is the full information maximum likelihood (FIML). In this approach, the entire model is estimated in a single phase. In general, the FIML estimation is more efficient than multi-step estimation. Until relatively recently, software for joint, full-information maximum likelihood estimation of all the parameters simultaneously was not available. This case is no longer true; several computer programs are available for FIML estimation of nested logit models. The LIMDEP software has the capability of estimating nested logit models using the FIML approach. Therefore, the models presented in this study are all calibrated using the FIML estimation approach.

Model Estimation Results

The adopted structure consists of a two level-nested structure as illustrated in Figure 1. It is already mentioned earlier that in the primary nest, total trips are divided into MV and NMV trips. In the secondary nest, the MV were split into autos(car/taxi), transit(bus) and CNG; and the NMV trips are split into rickshaw and walk trips. The trip data has been used to calibrate the journey to work trips. The results of journey to work trips are shown in Table 1.

The following table presents the estimated parameters of nested logit model of journey to work trips. Going over the parameters of the model, the estimated coefficients on the travel time, travel cost and travel time variability variables related to level of service attributes have the expected negative signs and they satisfactorily passed the 0.01 and 0.1 level of significance. In terms of attributes related to socioeconomic, all are statistically significant except gender in this case.

Table 1: Nested logit mode choice model of journey to work trip.

Variables	Coefficients	t-value
Attributes in the utility functions (beta)		
Attributes related to socioeconomic characteristics		
Income	0.191	6.248*
Occupation (*dummy variable; 1 if employed in government sector, 0 otherwise*)	0.151	2.324**
Gender (*dummy variable; 1 if male, 0 otherwise*)	0.051	1.213
Age	0.185	3.981*
Attributes related to level of service		
Time	-3.412	-8.321*
Cost/fare	-1.131	-1.648***
TTV	-0.855	-7.325*
Mode specific constants		
Auto (car/taxi)	0	
Transit (bus)	0.124	2.025*
CNG	3.217	3.964*
Rickshaw	1.214	1.968**
Walk	-0.189	- 0.249
Attributes of branch choice equations (gamma)		
Attributes related to socioeconomic characteristics		
Income	0.906	7.305*
Occupation (dummy variable)	0.035	2.534**
Gender (dummy variable)	0.066	2.839**
Age	0.192	3.516*
Attributes related to level of service		
Time	-1.478	-11.753*
Cost/fare	-0.128	-1.224

TTV	-0.123	-3.433*
Nest constants		
NMV	0	
MV	2.598	5.214*
Inclusive Values (Log-sum Parametrs) [IV parameters, tau(j|i,l),sigma(i|l),phi(l)]		
MV	0.845	6.147*
NMV	0.412	3.128*
Goodness of fit measures		
Log likelihood at estimated model (LL(β))	-179.548	
Log likelihood at zero (LL (0))	-341.361	
Likelihood ratio index ρ2 = 1 – LL (β) / LL (0)	0.474	

* Significance at 99% level; ** Significance at 95% level; *** Significance at 90% level

For the alternative specific constants of the modes, mode constants only walk was failed to achieve the minimum required level of significance. But other mode constants have been passed the reasonable level of significance. In terms of dummy variable (gender), the sign is positive which means that female prefer NMV than MV for journey to work trip.

The log sum parameter (inclusive value) is a function of the underlying correlation between the unobserved components for pairs of alternatives in that nest, and it characterizes the degree of substitutability between those alternatives. The value of the log sum parameter should be 0 < log sum < 1 to ensure consistency with random utility maximization principles. The estimated log sum parameters are 0.845 and 0.412 for MV and NMV respectively which imply non-zero correlation among pairs and this provides a statistical validation of using the nested logit structure. The dissimilarity is high (degree of dissimilarity) among the motorized vehicles within the nest than non-motorized vehicle as log sum of MV (0.845) is greater than log sum of NMV (0.412).

Considering the statistical measures of the model for journey to work trips, the obtained log-likelihood at convergence, *LL*(β*)* is -179.548 shows better performance as a model. The overall fit of the model is reasonable with a log likelihood ratio index (ρ^2) of 0.474 which means all variables included in the model are statistically significant.

Conclusion

Generally, the mode choice nested logit model is applied by a set of three model parameters. These model parameters include nesting coefficients, mode-specific constants, and level-of-service coefficients including socio-economic characteristics (i.e. coefficients of attributes). This paper describes the development of mode choice nested logit model for journey to work trip in Dhaka city. The calibration involved the travel time, travel cost, travel time variability, income, occupation, age and gender for journey to work trips. The selection of the proper universal nesting structure is critical to the development of a nested logit mode choice model. The nesting structure must address the existing transit service while at the same time provide suitable flexibility to permit the addition of future modes that might be considered. The selection of a nesting structure must also consider the data that are available for estimating the model. Several alternative nesting structures were investigated. Finally, the mode choice model was estimated as a two-level nested logit structure. This model included three MV modes and two NMV modes. The overall model was also calibrated using Full Information

Maximum Likelihood (FIML). Finally, in the estimated results income in socio-economic characteristics, travel time in level of service and CNG in mode choice preference are more sensitive to choose a particular mode for journey to work trip of city dwellers in Dhaka city.

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